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ABSTRACT

Mathematics faculty around the United States are using writing assignments in a variety of ways. A mathematics teacher at Alma College, Michigan, has students write mathematical autobiographies, keep a reading logbook, and write letters to other students about the course, letters to instructors about the topics, or about what they do not understand. Senior seminars at other institutions focus on famous equations, oral and written analyses of primary texts, or analysis of secondary texts. Calculus and pre-calculus lecturers also have found the usefulness of incorporating writing projects into their curricula. The writing assignments of a lecturer at Cornell University influence the structure of the dialogue he sets up with his students in a course entitled "From Space to Geometry." Serious mathematical and philosophical questions arise for the use of a seemingly trivial writing assignment--measure the height of a building and write the results as if a lab experiment were being explained. Discussions after completing the assignment usually lead to the realization that "mathematical methods" (such as Pythagorean Theorem) really do not work on a sphere such as the earth. Samples of students' writing throughout the course of the semester show how students can move from halting attempts to skillful use of language to explain mathematical concepts. Examples of other students' writing show the frustration that they can feel as they try to understand such concepts as a projection map. (RS)

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The Well-Tempered Mathematics Assignment

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I want to emphasize at the outset that I'm not on a writing faculty --- call it "truth in advertising," perhaps, or maybe I just want to avoid having to answer those really hard questions about "process and product" --- I'll admit at the start that I don't understand most of what's being said at this conference. What I AM is a mathematician, and what I will discuss is writing assignments in mathematics courses; mine, and other people's.

Now that I've said who I am, let me try to guess you who I think you might be. Some of you will be writing faculty who deal with writing across the curriculum, I assume; others will be directors of writing programs who have been making an attempt to get more science and mathematics faculty to use writing assignments in their classrooms; still others are faculty who have team-taught a course with a scientist or mathematician; one or two of you will have taught a math-with-writing course somewhere and will want to compare notes; and, I suspect, six or eight of you will be here for the next talk (hope I don't bore you). At any rate, what I expect you will be interested in is the following:

1. A compilation of some of the ways in which math faculty across the country have been using writing in their courses, perhaps so you can use it to convince some of your recalcitrant mathematicians to join the cause, or perhaps to get new ideas for some of the other faculty coming to you for help.
2. An outline of some writing assignments in a specific mathematics course, so as to provide a rationale for their use.
3. A student perspective on writing assignments in mathematics.

So first. let me mention some ways in which mathematics faculty around the US are using writing assignments and why they feel they are necessary.

Tim Sipka (ref. 9), at Alma College in Michigan, suggests that students write mathematical autobiographies; keep a reading logbook with questions; write letters to future students as to why the should take the course next term; write other letters to instructors about the topics, or about what the don't understand. or about the textbook. Tim also suggests some process papers, for instance:

1. You've often heard that division by zero is undefined. Explain why this is so. What are the mathematical reasons behind this rule?
2. Pretend you are the author of an encyclopedia of mathematics intended for both mathematicians and nonmathematicians. Write an appropriate entry for a term used in the course, for instance, for "derivative" in a calculus course.

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Meanwhile, Richard Montgomery at Southern Oregon State College (ref. 7) has a student seminar on famous equations. Dick's students choose a mathematical equation, such as:

$$x^n + y^n = z^n,$$
$$V - E + F = 2,$$
$$c = 2^{\aleph_0}.$$

They comb the library for information about the equation, which they then present to class in both a seminar talk and a term paper. Dick says his course is "overtly intended to counter the impression that mathematics somehow sprang full-grown into the bindings of textbooks."

Tom Barr of Rhodes College in Tennessee (ref. 1) has a slightly different senior seminar which emphasizes the enhancement of "oral and written communication." Tom chooses a primary text, for instance A. K. Dewdney's "The Turing Omnibus: 61 Excursions in Computer Science," and has the students design oral and written analyses of individual chapters and topics. They consult other references for definitions and examples, fill in missing details in the text, give examples to motivate the topics. About one week after students complete oral presentations, a written paper is due.

In another senior seminar at Southern Oregon, Dick Montgomery (ref. 6) has two "genies" (yes, genies), Alf and Bet make statements about the readings from the seminar. The students then pick one of the two statements and write a one to two page essay either endorsing or refuting the statement they chose. An example: After reading a Scientific American article about the 18th century French mathematician Sophie Germain, the students are asked to write on one of the following two statements:

Alf: In past centuries women who wanted to learn and do serious mathematics were given the same opportunity, recognition and respect as their male counterparts.

Bet: Searching for mathematical truth in the face of social obstacles requires a noble courage.

Senior seminars are not the only types of mathematics courses which have incorporated writing projects into their curricula; calculus and precalculus lecturers are also finding them useful.

John Meier of Lafayette College in Pennsylvania (ref. 4) gives students a "picture" of a function representing their height above the ground at time t during the day:

Then he asks the students to explain in writing what may have occurred to make the graph look the way it does.

In a later assignment, John asks students to find and clearly define functions which they have written into some of their descriptive passages. Still later, he has students incorporate functions as deliberate parts of an account of some scene from nature; for instance, they might talk about the functions derived from patterns the lights of cars make as they stream by windows of the students' rooms.

Roland Minton of Roanoke College in Virginia (ref. 5) gives mini-projects in calculus courses. Meanwhile, the Ithaca College Calculus Group (ref. 2) builds more extensive writing assignments into their calculus courses as follows: Every five weeks or so, the students are asked to work on a problem using the principles learned in the course. For instance, they might be asked to design a sports arena. After the students have worked together in small groups to investigate the stresses on the roof of the structure, say, they then are asked to write a report as to which design they thought most effective, why other designs were rejected, and what their reasoning was in the analysis.

Outside the standard calculus-oriented curriculum, student reports have often been used; for instance, in statistics classes. Robin Locke of St. Lawrence University in Upstate New York (ref. 3) has been using 1990 US government census statistics or baseball salary figures to have students write statistical analyses.

Let me now turn to some of my own involvement in writing.

I have spoken here before about my geometry course, From Space to Geometry, but let me describe a little of it again to outline how my writing assignments influence the structure of the dialogue which I set up with students.

I start my course by asking the students to recall geometric words they have used or studied. After a few minutes we get around to discussing the meaning of the word "geometry." I ask for a "working definition," one to use as a starting point for our explorations. Usually the students agree that "Earth-measurement" is a good working definition of geometry.

Now I give the students their first writing assignment: Go out to the Arts Quad to measure the height of the tallest building there. I tell them to write up their results as if they were explaining a lab experiment to me.

The students come back to class, and we have an animated discussion of what they did and how they did it. As our talking goes on, I note on the board the measuring techniques they used. These techniques tend to be of three types -- estimation ("My roommate stood next to the tower..."), physical ("...then I threw the stone..."), and mathematical ("I used Pythagoras," or "...similar triangles"). While I get a beachball, I remind them that we have tentatively decided that geometry is Earth-measurement. We now use the beachball to discover that their "mathematical methods," namely, the Pythagorean Theorem and the similar triangles method, don't work on the surface of the Earth. By the end of this discussion, we are really ready to begin the course.

My reason for describing my first assignment here is not necessarily to convince you to teach such a course (although it is enjoyable), but to show you how you can introduce some serious mathematical and philosophical questions by using a seemingly

trivial writing assignment: After all, what is "straight" on a beachball? How do you measure angles there? How do you define the word "triangle" on a sphere? What do we really do when we measure something? What assumptions do we make when we "geometrize?" And again, what is geometry, anyway?

Next, I will turn to the students themselves, through samples of their writing.

My first example is a very early paper from a student I'll call "Chris," for reasons you'll see in a while. Chris was a first semester freshman when he wrote this - note the date on the manuscript. Also, especially note the stilted language - the way in which he's trying hard to fit the language to the assignment.

SEPT. 20

Math 150 – Assignment 2

Go to a place you like. Sit down and describe it in writing. Compare it to a place you don't like. (You don't have to go to the place you don't like.)

Now notice all the geometric words you have used in your description. We will be discussing these words along with the place you have chosen and what they might have to do with each other in class next time.

IT IS VERY COMFORTABLE. IT IS QUIET. I AM LOOKING DOWN ON THINGS I AM ON A HILL. THERE IS A BIG, OPEN SPACE IN FRONT OF ME. I CAN SEE A RANGE FROM THE OPEN SEA TO THE FLAT LAND – A LAKE. THERE ARE MOUNTAINS IN THE BACKGROUND, BIG CURVES. I ONLY SEE THEIR SILHOUETTES. IN THE FOREGROUND THERE ARE MANY COLORS OF SHAPES – OF BUILDINGS & OF STRUCTURES, A ROAD RUNS PARALLEL TO WHERE I AM LOOKING THE MOUNTAINS EXTEND IN A PERPENDICULAR FASHION THE SPACES BETWEEN TREES & BUILDINGS BECOME EVEN MORE NARROW AS I LOOK FURTHER IN THE DISTANCE. LITTLE HILLS & BUMPS THAT STICK OUT FROM THE FLAT SURFACE OF THE EARTH.

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Let's now look at a second assignment, from about six weeks later. Chris is now working with the language to express his ideas - they aren't fully formed yet, but he's on his way. He's now trying to cope with and assimilate the ideas of the geometry course into the assignment; he's beginning to use a mathematical language to explain himself.

"Nemerov's "Figures of Thought" seems more symmetrical than Stevens' poem. It acknowledges man's ability to function effectively & progressively in a world of three-dimensions as opposed to an insect's inability. Stevens' poem depicts man as a victim to the overbearings of nature - nature is portrayed as the primary force. "Figures of Thought", however, depicts of honors man's keen sensibility to cipher in forms diverse & otherwise." Nemerov implies that mathematics lie in our perception of nature.

Humans blindly live in a world that involves mathematics. Shapes & sizes, perspective, & perception all have roots in mathematics. It is intriguing to find poets, consciously or not, finding the relationships between life & mathematics.

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Now here's an excerpt from Chris's final project on Christopher Columbus. See how skillfully Chris now navigates with geometric to explain Columbus's navigation through the world.

there was no way to precisely determine other elements of navigation - like speed. It wasn't until the sixteenth century that there was an accurate method to calculate speed. Initially, a light billet of wood was used to float in the water, attached to a line with knots. The knots were spaced so that as they were tossed overboard every half a minute, they equaled the number of nautical miles per hour the ship was moving. Thus, today we use the term knots in regard to speed at sea. Columbus, however, did not use this method. He simply estimated the speed by eye, using the passing seaweed and bubbles as indicators for speed. Usually, Columbus was fairly accurate considering the difficulties of this method. However, on his first returning voyage his calculations were way off. He had switched vessels, returning on the Nina as opposed to the Santa Maria. Columbus neglected to take this into account, which in turn threw his measurements of speed and distance off by a tremendous amount. - Discuss the effect this had on the voyage (secrecy, timing of landings, etc.)

Another adversity Columbus confronted upon his homeward voyage was the facing wind. He was forced to alter his route, though he had to keep careful records, so he would not lose track of direction. In order to do so, Columbus used a charting table called the traverse table. Columbus biographer, Samuel Morison, compares the traverse table to the "taxi-cab geometry" of New York City. "(Its) principle is to transform any number of diagonal courses into one big right angle; so many miles east or west (departure), and so

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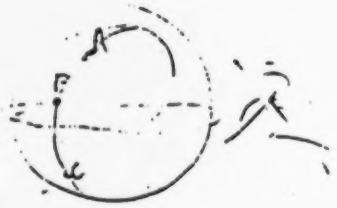
many miles north or south (difference of latitude). " (Morison, 193) This method proved to be a very efficient way to navigate into winds, and allowed Columbus to maintain accurate records and logs.

Columbus kept two separate logs throughout his voyages. One contained false records that were attainable by the crew. These records gave shorter distances than they had actually journeyed - deliberately lowered about twenty per cent the distance that Columbus had actually thought they had travelled. He used this tactic to not discourage his crew. The other log held the "true" records. It is ironic, however, that because Columbus had greatly overestimated the size of the earth, the "phony" records were much more accurate than the "true" records. It is this sense of manipulation that sets the conflict of the portrayal of Columbus's character today. He was not honest, just very ambitious. He was successful, though, and it is this success that - from an historical point of view - we must analyze and interpret. We must try to understand Columbus for what he stands for today - perhaps the world's greatest explorers. His navigational skills, evidently, were restricted. However, it was his keen mind and ambition that guided him overseas, and led him to the great discovery of the Americas.

As another example, let's see a paper from "Madeline." She's thinking out loud --- and doing rather well at it. What she's trying to get to is the idea of a projection map; she wants to take the Euclidean universe and map it (just as this transparency is being mapped) onto the surface of the Earth. If she can be led to this idea, she will actually be a long way toward an intuitive understanding of a great deal of modern mathematics and physics; for instance, hyperbolic space, geodesics, spacetime, and the like. I'd love to say that the class got her there, but unfortunately, it did not. Madeline improved somewhat past this point, but then she got sick and didn't finish the course. It would have been interesting to get her to write a final project on projection mappings to see her progression, but alas....

ES,



Math 150 – Classroom Assignment

Last time I talked about a triangle on a sphere.

How would you describe my definition? Does that definition make sense to you? Why, or why not? Would you suggest an alternate definition, or do you believe none can be constructed? And why would it matter at all whether such a thing as a triangle on a sphere could exist, anyway?

(A)

How do you mean this word? I'm not quite sure I understand

Assuming Euclidean depiction is objective that which superimposed onto a flat surface from the spherical surface, would form a triangle on the flat surface.

This is a good intuitive way to picture a triangle.

Everything depends on which language mode of space you chose to depict the triangle in.

In what way are these words related?

Assuming Volumetric depiction is native

But aren't we doing 2-dimensional stuff?

then triangle may be connection of any 3 lines formed by planes intersecting the sphere

"one" what?

one cannot be the other you must be either Euclidean or Geodesic or translate so to the other viewpoint with the interpreted.

I use me wins you... wing here - I'm self-always sure of your meaning. Although you make sense subjective sense

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Here's an example from another course, History of Mathematics, from last year. This will give you an idea of how I approach the class and the final projects. This student, "Ernie," has been told, as have all the students, that he can hand in a rough outline of his final project "if he wants." I then ask Ernie if he is willing to share what he has written with the group. He's willing, so I hand out copies to all the students. We ask Ernie where he gets his "criteria for discovery." Ernie responds, "Oh, I thought 'em up. I like to think about these things, and I knew if I was going to write about the acceptance of complex numbers, while I was reading I might as well have an idea of what it is I wanted to talk about.".

4 criteria for a new discovery to be accepted:
1) A new discovery solves an important problem;
2) A new discovery is presented by a mathematician with prestige;
3) A new discovery is given a geometric meaning;
4) A new discovery unites previously unrelated concepts.

criteria

Evidence it was accepted:
1) The new idea generates new research;
2) Notation becomes standardized.

First reference to complex numbers 1484 by Chuquet. Disregards them completely.
Cardano's Ars Magna (1545), work with the quadratic.

$x^2-10x+16=0$ $\quad$ $25-16=9$ $\quad$ roots $5-\frac{10}{2}, 5+\frac{8}{2}$

$x^2-10x+40$ $\quad$ $25-40=-15$ $\quad$ $(25-\sqrt{15})(25+\sqrt{-15})$ $\quad$ $(25-15i)(25+15i)$ $\quad$ $25-15i^2=40$

Bombelli's manipulation of Cardano's formula.

$x^3+px+q=0$ as $x=\sqrt[3]{-\frac{p}{2}+\sqrt{(\frac{p}{3})^3+(\frac{p}{2})^3}}+\sqrt[3]{-\frac{p}{2}-\sqrt{(\frac{p}{3})^3+(\frac{p}{2})^3}}$

$x^3=15x+4$ $\quad$ $x=\sqrt[3]{2+\sqrt{-121}}+\sqrt[3]{2-\sqrt{-121}}$

Bombelli visited $x=4$ solves, but Cardano's formula gives a mess.

$\sqrt{2+\sqrt{-121}}=2+b\sqrt{-1}$ and $\sqrt{2-\sqrt{-121}}=2-b\sqrt{-1}$

Cristoff Rudolff (1525), German, introduces the radical sign. Albert Girard (1629) first to use the radical with a negative number.
Descartes (1637) coins the term imaginary, makes distinction between negative and complex roots.
John Wallis in Algebra (1673) is first to give complex numbers geometrical meaning.
1702 John Bernoulli concludes that imaginary logarithms express real circular sectors.
Taken from Analysis of the Infinite

$\frac{1}{1+z^2}=\frac{\frac{1}{2}}{1+iz}+\frac{\frac{1}{2}}{1-iz}$

$\tan^{-1}$, stated by Leibniz.

D'Alembert Argued

Euler produces i for the square root of -1 in 1777. Not published into after his death.
Then Gauss makes the symbol popular.
Evidence of thinking of complex numbers as a plane.

$x^n-1=0 \quad \cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}$

Gauss in 1831 gives a geometrical representation of complex numbers that explain addition and multiplication. Gauss coins the word complex numbers.

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Here's page one of Ernie's final draft.

Although complex numbers appeared in various situations throughout the 16th and 17th centuries, they were not accepted as valid mathematical objects by the mathematical community during this time. In this paper, I will try to answer the question/ what were the prerequisites needed before complex numbers were accepted? En route ~~In root~~ to this goal, I will examine the discoveries that led mathematicians to the acceptance of complex numbers.

Before started my research, I developed four theories on how a new discovery becomes a mathematical object accepted by the mathematical community.

1) A new discovery ~~is~~ solves an important problem;
2) A new discovery is presented by a mathematician with prestige;
3) A new discovery is given a geometric meaning;
4) A new discovery unites previously unrelated concepts.

These four criteria were all present before the acceptance of complex numbers. It is partly conjecture to attribute the relative importance of these four criteria to the acceptance of complex numbers. I believe that it is for this reason that the authors of history of mathematics generally ignore this question and concentrate on such mundane issues as who published their paper first. As I examine the various discoveries that paved the way for complex numbers, I will analyze the importance of these discoveries the context of the four criteria listed above. As evidence that an idea is accepted by the mathematical community I propose two criteria;

For which reason?

1) The new idea generates new research;
2) Notation becomes standardized.

For the purposes of this paper, I consider complex numbers to be accepted when these two criteria are satisfied. However, this does not mean that they were completely understood. In fact, major mathematicians have had serious misgivings about the philosophical basis of complex number after these two criteria were satisfied. This illustrates that the history of mathematical ideas is a continuum. Change does not

does it this contradict the topic you're via this? I.e., with, both?

I told you earlier how "Madeline" upset me so by not finishing the course. Now let me show you a final paper by a student who could have been Madeline's twin sister, "Clementine." I'll let this one speak for itself.

Two lines l and m may be considered *parallel* if and only if (1) they are coplanar, and (2) they do not intersect (i.e. if no point lies on both of them) or they are the same line. If l and l` are lines of ∂ we say that l and l` are *parallel* if and only if l=l` or l and l` have no common point. This definition for parallelism does <u>not</u> say that l and m are <u>equidistant</u>, but merely that they will never cross. (Greenberg, p. 17) Specifically, Euclid defined parallelism as follows: parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction. (Faber, p. 126)

The definition itself presents a problem in determining if any lines are actually parallel. We cannot verify empirically whether two lines meet, because we can only draw segments, not lines. We can extend the segments further and further to see if they meet, but we cannot go on extending them to infinity. Thus, we are forced to verify parallelism indirectly, by using criteria other than the definition. (Greenberg, p. 18)

Good

Even Euclid seemed to know this.

This confusion might add difficulty to the problem of determining the geometry of a camera lens. If one accepts certain objects as parallel, however, one could analyze the accepted "parallel lines" in a photograph and determine that they are parallel within a specific geometry. Does the camera portray the world (still unverified as any type of geometry) as Euclidean, hyperbolic, or spherical?

Euclid's Fifth (parallel) Postulate states the following: "If a straight line falling on 2 straight lines makes the interior angles on the same side

Now do you see why I wanted Madeline to finish?

There is no replacement for writing assignments in a course like mine --- no set of twenty-three homework problems gets the mathematics across the way student writing does.

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